Physics I
ISI B.Math
Backpaper Exam : July 13, 2016

Total Marks: 100
Time : 3 hours

Answer all the questions.

1. (**Marks = 12 + 4 + 4**)

An electron of mass m and charge $-e$ is moving under the combined influence of a uniform electric field $E_0\mathbf{j}$ and a uniform magnetic field $B_0\mathbf{k}$. [Recall that the Lorentz force on a particle with charge q is given by $\mathbf{F} = q\mathbf{E} + q(\mathbf{v} \times \mathbf{B})$] . Initially the electron is at the origin and is moving with velocity $u\mathbf{i}$.
(a) Show that the trajectory of the electron is given by

$$x = a(\Omega t) + b\sin\Omega t,\ y = b(1 - \cos\Omega t),\ z = 0$$

where $\Omega = \frac{eB_0}{m}$, $a = \frac{E_0}{\Omega B_0}$ and $b = \frac{(uB_0 - E_0)}{\Omega B_0}$.

(b) Is the energy of the particle described in part (a) conserved ? Is the angular momentum about the origin conserved ? Justify your answers. Now consider the same system with the electric field turned off. Does this change the answers to the previous two questions ?

(c) In part (a) if the electric field is turned off and initially the electron is at the origin moving with a velocity $u\mathbf{i} + w\mathbf{k}$ how will the trajectory in (a) get modified ?

2. (**Marks = 5 + 5 + 10**)

(a) A billiard ball with speed v approaches an identical stationary one. The balls bounce of elastically, in such a way that the incoming one gets deflected by an angle θ and the stationary one gets deflected by an angle ϕ. Show that $\phi + \theta = \frac{\pi}{2}$

(b) In an elastic collision between an electron with kinetic energy E and an electron at rest, the incoming electron is observed to be deflected through an angle of $30°$. What are the energies of the two electrons after the collision?

(c) A mass M, initially moving at speed V, collides and sticks to a mass m,initially at rest. Assume $M \gg m$, and work in this approximation. What are the final energies of the two masses and how much energy is lost to heat, in :(i) the lab frame (ii)the frame in which M is initially at rest ?

3. (**Marks = 5 + 5 + 10**)

Consider an "elastic pendulum": A particle of mass m is attached to an elastic string of stiffness K and unstretched length l_0. Assume that the mass moves in a vertical plane.

(a) Write down the Lagrangian of the system choosing appropriate generalized coordinates . Are there any cyclic coordinates ? Is the total energy for the system conserved ? Is the angular momentum about the suspension point conserved ?

(b) Find the Lagrange equations for this system from (a) .

(c) Solve the Lagrange equations under the approximation of small angular and radial displacements from equilibrium. Your solution may contain arbitrary constants to be determined from initial conditions.

4. (**Marks** $= \mathbf{4 + 6 + 3 + 4 + 3}$)

Consider the motion of a particle of mass m under the influence of a force $\mathbf{F} = -k\mathbf{r}$ where k is a positive constant and $\mathbf{r}$ is the position vector of the particle.

(a) Show that the motion of the particle lies in a plane.

(b) Find the position of the particle as a function of time, assuming that at $t = 0, x = a, y = 0$ and $v_x = 0, v_y = v_0$.

(c) Show that the orbit is an ellipse.

(d) Find the period of motion.

(e) Does the motion of the particle obey Kepler's Laws of planetary motion ?

5. (**Marks** $= \mathbf{10 + 10}$)

(a) A cylinder of mass m, radius r and moment of inertia $I = \frac{1}{2}mr^2$ rolls without slipping down a plane inclined at an angle θ. What is the acceleration of the centre of the cylinder ?[Hint: use conservation of energy]

(b) A stick of length l slides perpendicular to itself (without rotating) across a frictionless horizontal table and collides elastically at one of its ends with a stationary ball. Both stick and ball have mass m. The mass of the stick is distributed in such a way that the moment of inertia around the CM, (which is at the centre of the stick) is $I = Aml^2$, where A is some number. What should A be such that the ball moves at the same speed as the centre of the stick after the collision ?